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A FIRST APPROACH ON TESTING NON-CAUSALITY WITH SYMBOLIC TIME SERIES

Abstract. *A novel non-causality test is developed based on the symbolic time series approach. The test is suggested to be complementary to the Granger non-causality test widely apply in empirical research. A statistic is constructed and its asymptotic distribution is derived. Size and power experiments are conducted comparing the results with the classical Granger non-causality. It is found that the symbolic test presents a good performance detecting nonlinear processes such as exponential, NLAR model and the chaotic Lorenz map.*
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JEL Classification: C12, C14

1. Introduction

Detecting causes and effects among variables has been an important topic not only in economics but also in the fields of statistics, artificial intelligence, philosophy, cognitive science, and the health and social sciences.

The concept "causality" has been discussed by many philosophers of mathematics. Russel (1913) asserted that the phenomenon "causality" did not exist in mathematics and physics. He claimed that causal relationship and physical equation are incompatible and the former should have been ruled out from sciences. On the other hand, there are philosophers such as Pearl (2009) defending the concept. In fact, he reviews all the approaches to cause and effect inference developed until now, showing that the concept is still alive in sciences.

As asserted by Hlaváčková-Schindler et al. (2007) a neutral definition of causality is hard to provide, since every aspect of causation has received substantial debate. However, in causation a characteristic remains, it is presumed that the cause chronologically precedes the effect. In fact, Granger (1969) remarks that there is little use in the practice of attempting to discuss causality without introducing time. Granger (2003) identifies two components of statement about causality: 1) the cause occurs before the effect; 2) the cause contains information about the effect that is unique, and is in no other variable.

There are different approaches that have been applied to analyze causality. In fact, it has been modeled by applying and combining mathematical logic, graph theory, Markov models, Bayesian probability among others. Hlaváčková-Schindler et al. (2007) make a review focusing on the information-theoretic approaches.

Schreiber (2000) introduces a measure quantifying causal information transfer between systems evolving in time, based on appropriately conditioned transition probabilities. In fact, the so-called Transfer Entropy is being applied in many fields. Smirnov (2013) studies spurious causalities with transfer entropy. He remarks that nowadays Transfer Entropy seems to be the most widely used tool to characterize causal influence in ensembles of complex systems from observed time series. However, he highlights that there are factors which might lead to spurious causality. In particular, he highlights three reasons: 1) unobserved variables influencing the system dynamics; 2) low temporal resolution; 3) observational noise.

Liang and Kleeman (2005) and Liang (2013) propose a line of work related with the information transfer. The authors assert that the Liang-Kleeman information flow provides a potential measure of the cause–effect relation between dynamical events, a relation usually hidden behind the correlation in a traditional sense.

The similarity index was introduced by Arnhold et al. (1999). Lungarella et al. (2007) assert that the major drawback of this index is that for weak causal structure and noisy time series, the detection of a coupling becomes difficult. The predictability improvement is another index that was introduced by Feldmann and Bhattacharya (2004). According to Lungarella et al. (2007) this index is close to Granger causality because it estimates the regression errors from state vectors. The difference is the lack of autoregressive modeling.

The linear framework for measuring and testing causality has been widely applied in economics and finance. In particular, the Granger non-causality test based on application of bivariate autoregressive models has been extensively applied in economics since developed in seminar papers by Wiener (1956), Granger (1969) and Sims (1972). They introduced a specific notion of causality into time series analysis and nowadays this test is found in most of statistical and econometrics software. Even if the linear approach is widely applied nowadays, there are recent developments in nonlinear Granger causality such as Baek and Brock (1992), Diks and Panchenko (2006), Kyrtsou and Labys (2007) and Hristu and Kyrtsou (2010). However, as remarked by Hlaváčková-Schindler et al. (2007) linear and nonlinear model based Granger causality approaches present some problems. Indeed, the selected model must be appropriately matched to the underlying dynamics; otherwise model misspecification would arise, leading to spurious causality values. They remark that a suitable alternative would be to adopt non-parametric approaches which are free from model mismatch problems. In this sense, in the present paper, a novel nonparametric approach based on symbolic time series is introduced. The objective is to develop a simple, easy to compute and powerful test. In fact, as will be shown the present test has the advantage of not depending on any type of underlying model specification. Besides, it is expected to gain robustness in the presence of noise by applying symbolic time series analysis.

It is well known that noise is frequently met in real world leading to spurious causality as remarked by Smirnov (2013).

The paper is organized as follows. The section 2 presents the symbolic time series approach. Section 3 develops the symbolic non-causality test. In section 4 size and power experiments are conducted. In section 5 some empirical data are tested applying the new test comparing the results with the Granger Non-causality test. Finally, section 6 draws some conclusions.

2. Symbolic Time Series Analysis

As mentioned by Finney et al. (1998) the concept of symbolization has its roots in dynamical-systems theory, particularly in the study of nonlinear systems which can exhibit bifurcation and chaos. As mentioned in the previous section, besides the computational efficiency, symbolic methods are also robust when noise is present. Williams (2004) highlights that symbolic dynamics is a method for studying nonlinear discrete-time systems by taking a previously codified trajectory using sequences of symbols from a finite set also called alphabet. However, as Piccardi (2004) remarks symbolic dynamics should be differentiated form symbolic analysis. The former denotes theoretical investigation on dynamical systems. The latter is suggested when data are characterized by low degree of precision. The idea in symbolic analysis is that by discretizing the data with the right partition we obtain a symbolic sequence. This sequence is able to detect the very dynamic of the process when data are highly affected by noise.

Data symbolization implies transforming an original series of measurements into a limited number of discrete symbols. The resulting symbolic series can be analyzed for nonrandom temporal patterns. It means that given a time series $\{x_t\}_{t=1,2,\ldots,T}$, we study the dependence present in the series by translating the problem into a symbolic time series $\{s_t\}_{t=1,2,\ldots,T}$.

Let us consider a time series $\{x_t\}_{t=1,2,\ldots,T}$ where T is the sample size. Symbolic Time Series Analysis (STSA) approach suggests as a first step to take a partition such that the individual occurrence of each symbol is equiprobable with all others. The result is $\{s_t\}_{t=1,2,\ldots,T}$ a symbolized time series. For instance, imagine $\{x_t\}_{t=1,2,\ldots,T}$ is a time series generated by a Gaussian white noise, we can define a discretization of two regions by establishing $s_t=0$ when x_t takes a value in the first 50% of the density function and $s_t=1$ in the other case. The new discrete time series of events $\{s_t\}_{t=1,2,\ldots,T}$ would be similar to a series generated by tossing a coin. Of course different discretization could be applied, for example six equally likely symbols could be interpreted as tossing a die.

3. Symbolic Non-Causality Test

In the present section a new non-causality test is introduced. Applying the STSA approach explained in the section before, a new statistic is proposed and the asymptotic distribution is derived.

The main idea is to derive an asymptotic distribution for the statistic when there is no causality between the series. As a first approach two independent random time series sized $T+1$ will be considered.

Assume that X and Y are two independent random time series sized $T+1$. We can define a partition in "a" equiprobable regions obtaining two symbolized time series $Sx=\{sx_1,sx_2,..,sx_{T+1}\}$ and $Sy=\{sy_1,sy_2,\dots,sy_{T+1}\}$.

Once the time series are symbolized we have to establish the two hypotheses we want to test: 1) Sx does not cause Sy; 2) Sy does not cause Sx. Let us define two new series grouping the two time series in the following way:

1) $Sxy=\{(sx_1, sy_2), (sx_2, sy_3),\dots,(sx_{t-1},sy_t),\dots,(sx_T,sy_{T+1})\}$
2) $Syx=\{(sy_1, sx_2), (sy_2, sx_3),\dots,(sy_{t-1},sx_t),\dots,(sy_T,sx_{T+1})\}$

Note that sx and sy takes values from an alphabet A composed by "a" symbols. Therefore the combination (sx_{t-1}, sy_t) takes a value from a set of $n=a^2$ possible events. For instance if $a=3$, the combination will takes values from the set $\{(1,1), (1,2), (1,3), (2,1), (2,2),(2,3),(3,1),(3,2),(3,3)\}$ having 9 elements. Since Sx and Sy are assumed random and independent no event should be more probable and each event should have probability $1/a^2$ indicating non-causality. However, in case that one of these events is more probable there is some evidence of causality. For instance, imagine that with $a=3$, the event $(1,3)$ is more likely, this mean that each time that sx takes a value 1 in time $t-1$, sy takes value 3 in time t. Note that pairing the variables two by two, one observation is lost and Sxy and Syx are sized T.

Note that, defining Exy_i for $i=1,\dots,n$ as the sum of the total i events in the set Sxy and Eyx_i for $i=1,..,n$ as the sum of the total i events in the set Syx, we can derive two multidimensional variables $Exy=\{Exy_i/T\}$ and $Eyx=\{Eyx_i/T\}$. The two variables will follow a multinomial distribution function.

Exy is distributed multinomial with $E(Exy_i)=(1/n)$, $Var(Exy_i/T)=(1/n)((n-1)/nT)$ and $Cov(Exy_i, Exy_j)=-(1/n)(1/nT)$ for all $i\neq j$. Similarly, Eyx is distributed multinomial with $E(Eyx_i)=(1/n)$, $Var(Eyx_i/T)=(1/n)((n-1)/nT)$ and $Cov(Eyx_i, Eyx_j)=-(1/n)(1/nT)$ for all $i\neq j$.

Since a multinomial distribution can be approximated by a multivariate normal distribution, it is possible to say that Exy_i/T and Eyx_i/T will follow $N(1/n,\sigma^2\Omega)$ where σ^2 is $(1/nT)$ and Ω is a idempotent matrix as in (1).

$$\Omega_{nxn} \equiv \begin{bmatrix} (n-1)/n & -1/n & \dots & -1/n \\ -1/n & (n-1)/n & \dots & -1/n \\ \dots & \dots & \dots & \dots \\ -1/n & -1/n & \dots & (n-1)/n \end{bmatrix} \qquad (1)$$

For convenience we can define the vector variable $\{\varepsilon xy_i\}=\{(Exy_i/T)-(1/n)\}_{i=1,2,\ldots,n}$ having a multivariate normal distribution $N(\o,\sigma^2\Omega)$, being $\o$ the null vector. Similarly, we can define the vector variable $\{\varepsilon yx_i\}=\{(Eyx_i/T)-(1/n)\}_{i=1,2,\ldots,n}$ having a similar multivariate normal distribution $N(\o,\sigma^2\Omega)$. Then the statistic for the both hypothesis can be defined as in (2) and (3).

$$\left\{\frac{\sum_{i=1}^{i=n}\varepsilon xy_i^2}{\sigma^2}\right\} \tag{2}$$

$$\left\{\frac{\sum_{i=1}^{i=n}\varepsilon yx_i^2}{\sigma^2}\right\} \tag{3}$$

The term in brackets in (2) and (3) are quadratic forms in random normal variables. As Mathai and Provost (1992) assert, the distribution of quadratic forms in normal variables has been extensively studied by many authors. Various representations of the distribution function have been derived and several different procedures have been given for computing the distribution and preparing appropriate tables.
In the present paper the following theorem in Mathai and Provost (1992, page 197) is applied:
The necessary and sufficient condition for quadratic form *X'AX* to be distributed as a chi-square varies with *r* degrees of freedom when *X* has a multivariate normal distribution with mean vector $\o$ and possibly singular covariance matrix Ω, are:

(i) $(A\Omega)^2=(A\Omega)^3$ and $tr(A\Omega)=r$

(ii) $tr(A\Omega)2=tr(A\Omega)=r$ and $\rho(\Omega A\Omega)=r$

Note that the theorem can be applied in the present case where vector $X=(\varepsilon_1/\sigma,\varepsilon_2/\sigma,\ldots,\varepsilon_n/\sigma)$ is distributed multivariate normal $N(\o,\Omega)$. In this case *A* is the identity matrix *I* and Ω is symmetric, singular and idempotent. Since $tr(A\Omega)=n-1$, thus *X'AX* distributes Chi-square with $(n-1)$ degrees of freedom.
Remembering that $\sigma^2=(1/nT)$ then we obtain that the distribution of the Symbolic Non-Causality test (SNC) as in (4) and (5).

$$SNC(X \rightarrow Y) \equiv nT\left\{\sum_{i=1}^{i=n}\left(\frac{Exy_i}{T}-\frac{1}{n}\right)^2\right\} \textit{assymptotically distributes } \chi^2_{n-1} \tag{4}$$

$$SNC(Y \to X) \equiv nT\left\{\sum_{i=1}^{i=n}\left(\frac{Eyx_i}{T}-\frac{1}{n}\right)^2\right\} assymptotically\ distributes\ \chi^2_{n-1} \quad (5)$$

We derive the test assuming that X and Y are random processes. However, most of the time the processes are autocorrelated and this effect should be mitigated to not affect the result of the test. A first approach is applying an autoregressive process and testing the non-causality between the residuals of the two series.

$$x_t=\alpha_0+\alpha_1 x_{t-1}+ux_t \quad (6)$$

$$y_t=\beta_0+\beta_1 y_{t-1}+uy_t \quad (7)$$

Note that in practice computing the statistic is very simple. In summary, the test works as follows:

Step 1: Consider time series $\{x_t\}_{t=1,2,\dots,T+2}$ and $\{y_t\}_{t=1,2,\dots,T+2}$ and apply an AR(1) to both series as in (6) and (7) in order to eliminate autocorrelation and define the new residuals time series $\{ux_t\}_{t=1,2,\dots,T+1}$ and $\{uy_t\}_{t=1,2,\dots,T+1}$.Note that 1 observation is lost after applying AR(1).

Step 2: In $\{ux_t\}_{t=1,2,\dots,T+1}$ and $\{uy_t\}_{t=1,2,\dots,T+1}$ apply a partition in a equiprobable regions and translate the series into $\{sx_t\}_{t=1,2,\dots,T+1}$ and $\{sy_t\}_{t=1,2,\dots,T+1}$.

Step 3: According to the two hypothesis $X\to Y$ and $Y\to X$ define the two sets $Sxy=\{(sx_1,sy_2),(sx_2,sy_3),\dots,(sx_{t-1},sy_t),\dots,(sx_T,sy_{T+1})\}andSyx=\{(sy_1,sx_2),$ $(sy_2,sx_3),\dots,(sy_{t-1},sx_t),\dots,(sy_T, sx_{T+1})\}$.

Step 4: For Sxy and Syx compute the frequency of the $n=a^2$ different events Exy_i/T for $i=1,2,\dots,n$ and Eyx_i/T for $i=1,2,\dots,n$.

Step 5: For Sxy and Syx compute the $SNC(X\to Y)=nT\{\Sigma[(Exy_i/T)-(1/n)]^2\}$ and $SNC(Y\to X)=nT\{\Sigma[(Eyx_i/T) - (1/n)]^2\}$ as shown in equations (6) and (7).

Step 6: Compare the $SNC(X\to Y)$ with the Chi-2 with n-1 degree of freedom at 0.05 of significance, under the null hypothesis that X does not cause Y. When $SNC(X\to Y)$ is larger than the critical value we reject the null hypothesis. Similarly when $SNC(Y\to X)$ is larger than the critical value we can reject that Y does not cause X.

In the following section and for the sake of simplification and trying to minimize the tradeoff between alphabet size and sample size, we will consider 3 symbols. An alphabet of $a=3$ symbols determines $n=32=9$ possible events in the set of pairs $\{(x_{t-1},y_t)\}$ *or* $\{(y_{t-1}, x_t)\}$. In this case the 9 frequencies Exy_i/T and Eyx_i/T could be approximated by a multivariate normal distribution $N(1/9,\sigma^2\Omega)$ where σ^2 is $(1/9T)$ and Ω is a idempotent matrix as in (8).

$$\Omega_{9x9} \equiv \begin{bmatrix} 8/9 & -1/9 & \dots & -1/9 \\ -1/9 & 8/9 & \dots & -1/9 \\ \dots & \dots & \dots & \dots \\ -1/9 & -1/9 & \dots & 8/9 \end{bmatrix} \quad (8)$$

The *SNC(X→Y)* and *SNC(Y→X)* are defined as in (9) and (10).

$$SNC(X \rightarrow Y) \equiv 9T\left\{\sum_{i=1}^{i=9}\left(\frac{Exy_i}{T}-\frac{1}{9}\right)^2\right\} \text{assymptotically distributes } \chi_8^2 \qquad (9)$$

$$SNC(Y \rightarrow X) \equiv 9T\left\{\sum_{i=1}^{i=9}\left(\frac{Eyx_i}{T}-\frac{1}{9}\right)^2\right\} \text{assymptotically distributes } \chi_8^2 \qquad (10)$$

4. Conducting Size and Power Experiments

The present section studies the performance in finite sample and the power detecting different forms of causality. The Granger non-causality test is applied as a benchmark in order to compare the introduced symbolic test.

It is important to remark to things. Firstly, the present exercise has to be conducted for small sample since economics time series are generally small and this is the most important consideration. This is true when considering annual time series like GDP, inflation rate or unemployment rate where dataset generally start around the year 1900 counting at most with 100 observations, monthly data could arrive to 1200. Secondly, the exercise takes the classical Granger non-causality test just as a benchmark in order to compare the results and since this test is widely applied most of the empirical research in economics. The objective will not be to suggest any superior consideration about the present test, at most it will be suggest that both tests could be complementary.

Granger non-causality test is constructed considering the following VAR model:

$$X_t=\alpha_0+\alpha_1X_{t-1}+\alpha_2Y_{t-1}+\varepsilon x_t$$
$$Y_t=\beta_0+\beta_1Y_{t-1}+\beta_2X_{t-1}+\varepsilon y_t$$

where εx_t and εy_t are independent and normal residuals. The significance of parameters α_2 and β_2 determines the rejection of non-causality from *Y* to *X* and from *X* to *Y*, respectively.

The Symbolic Non-Causality test is conducted taking the stationary time series and applying an AR(1) process for each series as shown in equations (6) and (7) and the residuals are symbolized and the *SNC(X→Y)* and *SNC(Y→X)* are computed as shown in section 3. If *SNC(X→Y)* is larger than the critical value at 5% of a Chi-2 with 8 degree of freedom, non-causality is rejected. The same is valid in the case of *SNC(Y→X)*.

The following experiment was conducted to study the SNC size. 10,000 Monte Carlo simulations were conducted for time series of pseudorandom Gaussian *i.i.d (0,1)* for independent variables *X* and *Y* and for different sample sizes ($T=50$, $T=100$, $T=500$, $T=1000$, $T=5000$). The tests were applied considering significance levels $a=0.01$, $a=0.05$ and $a=0.10$. Therefore we compute the percentage of null hypothesis rejection over the 10,000 Monte Carlo simulations for each hypothesis ($X \rightarrow Y$ and $Y \rightarrow X$) and for each test (SNC and Granger). When the critical values are unbiased, the rejection percentage should be near to the significance levels for non-causality.

Table 1 shows the percentage of the null hypothesis rejection for the case of non-causality. The first column shows the three applied significance levels (1%, 5% and 10%), the second columns refers to the five sample sizes. For the SNC test and the Granger Non Causality test there are two hypotheses referring to *X* non-causing *Y* and *Y* non-causing *X*. Note that Granger Non-Causality test correctly detects non-causality in any direction with rejection percentages to the significance levels. The Symbolic Non-Causality test is more conservative, the percentages of rejection are very low, in many cases less than 1%. Therefore, since the test seems to be very conservative rejecting non-causality more time than expected we should contrast its power detecting different types of causality.

Table 1. Size of the Symbolic Non Causality Test and the Granger Non Causality Test

Significance	**Sample Size**	**Symbolic Non Causality (SNC)**		**Granger Non Causality**	
		$X \rightarrow Y$	$Y \rightarrow X$	$X \rightarrow Y$	$Y \rightarrow X$
α=1%	*T=50*	0.32	0.16	0.90	0.92
	T=100	0.06	0.01	1.00	1.04
	T=500	0.05	0.07	0.91	1.10
	T=1.000	0.05	0.05	1.01	0.95
	T=5.000	0.06	0.10	1.12	1.06
α=5%	*T=50*	0.52	0.42	5.12	5.06
	T=100	0.43	0.30	5.24	5.07
	T=500	0.41	0.42	4.90	4.95
	T=1,000	0.35	0.45	4.83	4.78
	T=5.000	0.60	0.46	5.20	5.24
α=10%	*T=50*	0.87	0.85	10.10	10.25
	T=100	1.20	0.98	10.19	10.12
	T=500	0.99	1.07	9.60	9.88
	T=1.000	1.01	0.98	10.44	9.73
	T=5.000	1.16	1.14	10.11	10.73

***Note**: 10.000 Monte Carlo simulations were conducted applying the normal distributed pseudorandom numbers from MatLab R2010a. The values are the percentage of the null hypothesis rejection.*

The next experiment will study the power of the SNC test comparing with the well-known Granger non-causality test. Ten different processes were simulated including linear and nonlinear stochastic processes and deterministic and chaotic processes. The following is the list of the 10 processes:

1. AR(1). Two independent time series originated by autoregressive processes: $X_t=0.2+0.45X_{t-1}+\varepsilon_{1t}$ and $Y_t=0.8+0.5Y_{t-1}+\varepsilon_{2t}$. Where ε_{1t} and ε_{2t} are *i.i.d. normal(0,1)*

2. 1-VAR(1). $Y_t= 0.04-0.4Y_{t-1}+0.16X_{t-1}+v_{1t}$ and $X_t= 0.07+0.69X_{t-1}+0.5Y_{t-1}+v_{2t}$; where the vector (v_{1t},v_{2t}) is distributed multinomial with $(0,0)$ and variance and covariance matrix

$$\begin{pmatrix} 3{,}64 & -0{,}11 \\ -0{,}11 & 0{,}79 \end{pmatrix}$$

3. 2-VAR(1). Similar to 1-VAR(1) but X_t does not depend on Y_{t-1}: $Y_t= 0.04-0.4Y_{t-1}+0.16X_{t-1}+v_{1t}$ and $X_t= 0.07+0.69X_{t-1}+v_{2t}$; where the vector (v_{1t},v_{2t}) is distributed multinomial with $(0,0)$ and variance and covariance matrix

$$\begin{pmatrix} 3{,}64 & -0{,}11 \\ -0{,}11 & 0{,}79 \end{pmatrix}$$

4. Non-Linear with Exponential component. $X_t=1.4-0.5X_{t-1}e^{Y_{t-1}}+\varepsilon_{1t}$ and $Y_t=0.4+0.23Y_{t-1}+\varepsilon_{2t}$; where ε_{1t} and ε_{2t} are i.i.d. normal(0,1)

5. Non-Linear with Logarithmic component: $Y_t=0.1+ 0.7Log|X_{t-1}| +0.3Y_{t-1}+\varepsilon_{1t}$ and $X_t= 0.1+0.2X_{t-1}+\varepsilon_{2t}$; where ε_{1t} and ε_{2t} are i.i.d. normal(0,1)

6. ARCH(1): $\sigma^2_{xt}= 0.15+0.6x^2_{t-1}$; $\sigma^2_{yt}= 0.02+0.4y^2_{t-1}$; $x_t=0.20+\sigma_{xt-1}\varepsilon_{1t} + 0.6y_{t-1}$ and $y_t=0.05+\sigma y_{t-1}\varepsilon_{2t}$; where ε_{1t} and ε_{2t} are i.i.d. normal(0,1)

7. NLAR (Autoregressive Nonlinear): $X_t=0.2|X_{t-1}|/(2+|X_{t-1}|)+\varepsilon_{1t}$ and $Y_t=0.7|Y_{t-1}|/(1+|X_{t-1}|)+\varepsilon_{2t}$; where ε_{1t} and ε_{2t} are i.i.d. normal(0,1)

8. Henon: $X_t=1+Y_{t-1}-1.4X^2_{t-1}$ and $Y_t=0.3X_{t-1}$; with initial conditions Y_1 generated randomly by $|N(0,0.01)|$ and $X_1=1$

9. Lorenz: $X_t=1.96X_{t-1}-0.8X_{t-1}Y_{t-1}$; $Y_t=0.2Y_{t-1}+0.8X^2_{t-1}$; with initial conditions X_1, Y_1 generated randomly. This is a discrete version of the Lorenz process as in [29] Stork et al. (2009).

10. Semi-Qualitative model: $X_t=0.08+0.2Y_{t-1}X_{t-1}+0.1\varepsilon_t$; with the following conditions: if $X_{t-1}\geq 0.09$ then $Y_t=1$, if $0.08\leq X_{t-1}<0.09$ then $Y_t=0$; finally if $X_{t-1}<0.08$ then $Y_t=1$.where ε_t is *i.i.d.* normal(0,1).

Table 2 shows the results of the power experiments applying the SNC and the Granger Non Causality test to 10,000 Monte Carlo simulations for the ten models and for different sample sizes (T=50, 100, 500, 1000 and 5000).

Table 2.Power test for the SNC Test and the Granger Non Causality Test

Sample Size	Model	Symbolic Non Causality		Granger Non Causality		Model	Symbolic Non Causality		Granger Non Causality	
		$X\to Y$	$Y\to X$	$X\to Y$	$Y\to X$		$X\to Y$	$Y\to X$	$X\to Y$	$Y\to X$
T=50		0.40	0.45	5.66	5.25		0.55	1.56	5.28	22.55
T=100	**AR(1)**	0.42	0.37	5.28	5.09	**ARCH(1)**	0.27	3.00	5.64	39.82
T=500		0.41	0.27	5.47	5.14		0.40	46.75	5.51	94.53
T=1.000		0.39	0.42	5.18	5.14		0.51	90.00	5.63	99.78
T=5.000	(None)	0.34	0.46	5.30	5.09	(Y→ X)	0.58	100.00	6.08	99.99
T=50		5.07	1.88	84.39	41.46		0.01	0.01	0.05	0.05
T=100	**1-VAR(1)**	16.04	4.89	99.11	72.50	**NLAR**	0.73	0.34	4.82	4.74
T=500		99.21	76.49	100.00	99.99		5.96	0.31	4.94	5.16
T=1.000		100.00	99.48	100.00	100.00		17.87	0.29	5.01	5.05
T=5.000	(X→ Y , Y→ X)	100.00	100.00	100.00	100.00	(X→ Y)	98.02	0.39	6.51	5.00
T=50		5.06	0.42	83.59	4.65		100.00	3.43	100.00	2.06
T=100	**2-VAR(1)**	15.45	0.24	98.98	4.47	**Henon**	100.00	79.26	100.00	21.17
T=500		98.58	0.31	100.00	5.03		100.00	100.00	100.00	99.98
T=1.000		100.00	0.37	100.00	5.16		100.00	100.00	100.00	100.00
T=5.000	(X→ Y)	100.00	0.38	100.00	4.91	(X→ Y , Y→ X)	100.00	100.00	100.00	100.00
T=50		0.51	3.76	2.89	16.89		96.61	31.90	30.77	13.86
T=100	**Nonlinear Exponential**	0.28	11.85	2.78	13.36	**Lorenz**	99.99	90.49	28.52	12.64
T=500		0.43	89.50	2.53	11.48		100.00	100.00	23.60	11.74
T=1.000		0.40	99.22	2.73	11.29		100.00	100.00	24.42	11.69
T=5.000	(Y→ X)	1.42	100.00	2.67	11.19	(X→ Y , Y→ X)	100.00	100.00	23.87	11.52
T=50		15.77	0.37	9.64	5.57		100.00	26.60	100.00	11.34
T=100	**Nonlinear Logarithm**	54.04	0.25	12.01	4.93	**Semi-Qualitative**	100.00	68.85	100.00	25.74
T=500		100.00	0.48	34.21	5.17		100.00	100.00	100.00	90.89
T=1.000		100.00	0.25	56.85	5.06		100.00	100.00	100.00	99.65
T=5.000	(X→ Y)	100.00	0.39	99.59	5.08	(X→ Y , Y→ X)	100.00	100.00	100.00	100.00

***Note**: 10.000 Monte Carlo simulations were conducted applying the normal distributed pseudorandom numbers from MatLab R2010a. The values are the percentage of the null hypothesis rejection. The significance level is 0.05.*

In the present work as a criterion we consider 60% as a threshold when rejecting or no rejecting the null hypothesis. Non causality in model AR(1) is correctly detected by both tests and for all the sample sizes. Note in Table 2 that the SNC is still more conservative rejecting causality in both directions with a percentage less than 5%.

Model 1-VAR(1) and 2-VAR(2) differ just in one parameter generating bi-directionality in the first model and causality from X to Y in the second one. Note that both test detects the causality in 1-VAR(1) for sample sizes larger than 500. Granger test detects the causality for a sample of 100 in this case.

In the case of 2-VAR(1) both test detects causality from X to Y when sample size is larger than 500. Note that for small samples Granger test detects this process better than SNC. This may due to the fact that Granger test is based on the VAR model.

The nonlinear model with an exponential component implies causality from Y to X. Note that SNC detects the causality when the sample size is 500 or larger. However, Granger test does not detect causality in any case.

The fourth model is nonlinear with a logarithmic term generating causality form X to Y. In this case SCN detects the causality when size is 500 or larger. However, Granger test needs 5,000 observations to detect the causality.

An ARCH process with causality from Y to X is detected for T=1000 in the case of SCN and for T=500 for the Granger test.

As asserted by Risso (2014) the NLAR process is very difficult to detect. Note that SCN is the only one detecting the causality when T=5000.

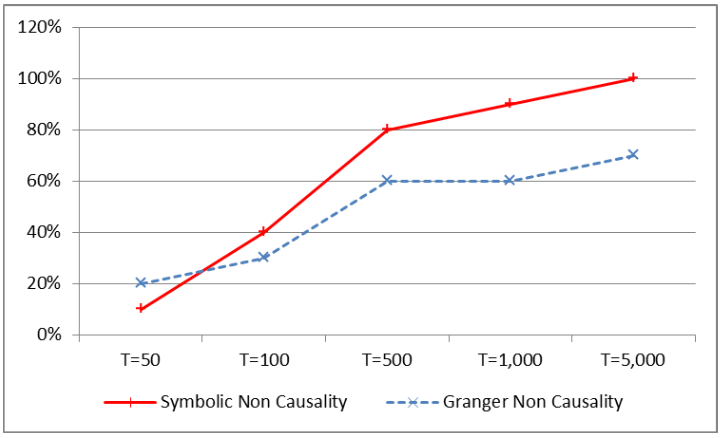
The Henon map is a chaotic process with causality in both directions. Note that SNC detects the process starting form T=100 but the Granger test needs a sample of 500.

The Lorenz discrete map is also chaotic and it is detected by SNC starting from T=100. However, note that Granger test never detects the causality.

The Semi-qualitative model is correctly detected by SNC starting from T=100 but Granger test needs 500 observations.

In summary, for a sample size of 5,000 the SNC test is able to detect causality for the 100% of the models. However, the Granger test detects 70% of the process. In particular, Granger test is not able to detect the nonlinear exponential model, the NLAR model and the Lorenz chaotic map.

Figure 1.Percentage of correct causality detection depending on the sample size



Note: Elaborated based on the Monte Carlo simulations.

Figure 1 shows the percentage of correct causality detection on the ten models depending on the sample sizes. Note that for a sample size of 50 the Granger Non causality test has a better performance than SNC detecting 20% of the process. However, starting with T=100 the SNC test shows a better performance until detecting all the process when T=5000. In nonlinear processes the best performance is shown by the SNC. In particular, is highlighted that Granger test is not able to detect the model with an exponential component, the NLAR model and the chaotic Lorenz map.

5. Non-Causality in Empirical Data

The objective of this section is to compare the results from the SNC test and the Granger test when are applied to empirical data. Note that working with empirical data, historical series, there is no certainty about the very data generator process due to the fact that most of the time the underlying process is unknown. On the other hand, usually the samples are contaminated by noise and this may generate spurious causality or not to permit the detection of causality.

In the present work a set of 22 US stocks and 2 indices where selected considering weekly and monthly frequencies. Stocks returns from September 1972 to May 2014 where collected for the following stocks: AA, AXP, BA, CAT, CVX, DD, DE, DIS, GE, GT, HPQ, IBM, JNJ, KO, MCD, MMM, MRK, PFE, PG, WFC, WMT, XOM. In addition, the indices GSPC (S&P 500) and NASDAQ were included. Weekly data from the last week of August 1972 to the last week of May 2014 were considered for the same set of returns. In summary, 24 times series from US stock market where tested counting with 501 monthly data and 2178 weekly data.

Table 3 shows the results of the Symbolic Non Causality test applied to the 24 financial returns paired two by two totaling 576 tests. The first cell of the first column indicates the testing direction going from the variable in the first column to the variable in the first row. The names of the variables in the first columns are the variables that are considered to cause the variables in the titles of the first row. For instance, causality test from CVX to DD shows the statistic 13.39 which is larger than the Chi2 at a significance level of 10% (13.36) then the non-causality is rejected and it could be affirmed that CVX causes DD. However, the test suggests that there is not causality from DD to CVX because the statistic is 2.42.

It is highlighted that 19 causalities are detected by the test. In particular, note that NASDAQ seems to be caused by 3 different stock returns, the S&P 500 index and the NASDAQ past. DD and DIS are also caused by 3 different returns. On the other hand, note that CVX is the most affecting return in the matrix, causing 4 series.

Table 4 presents the Granger non-causality test for the same 501 monthly data. Note that 84 causalities are suggested by this test. In particular, it is remarkable that Boing (BA) is caused by nine stocks and the two US indices. On the other hand, Wal-mart (WMT) is causing 9 US stocks according to the Granger test.

It seems that Granger non-causality test detects more causality relationships than the introduced SNC. However, as mentioned before it should be considered that when working with empirical data we do not control de process. Even more we do not know the real process behind and we may not being considering unobserved variables. It may be happen that some relationships be spurious. On the other hand, it is interesting to remark that some causality relations detected by SCN are no detected by the Granger non-causality test. In particular, CVX causes DD and DIS according to SNC but there is no causality according to the Granger test. PG is causing AXP and KO when applying the SNC but Granger test suggests that is causing GT. In this sense, it could be suggested to apply both tests as complementary.

Table 5 shows the results of the SNC test applied to the 2178 weekly data. In this case the test detects 51 causality cases. In particular, is highlighted that S&P 500 is causing 8 US stocks and WFC is caused by 9 US stock and S&P500 index.

Table 6 shows the Granger non-causality test for the same dataset. Note that 129 causality relations are suggested by the test. MCD is causing 13 stocks and the two US indices. CAT is caused by 12 US stocks and the 2 indices.

Comparing Table 5 and Table 6 it could be remarked again that there are some causality relations suggested by SNC but not detected by the Granger causality and vice versa. For instance, XOM is causing AA according to SCN but there is not causality when Granger non-causality is applied.

As mentioned before, with working with empirical data spurious causality may arise. Non stationary time series is remarked as an important cause of spurious non-causality. However, in the present case US stock returns are clearly stationary time series. A second motive for spurious non-causality is highlighted by Nalatore et al. (2007). They assert that when data is contaminated by noise spurious causality between two measured variables can arise and true causality can be suppressed. In this sense, symbolic time series analysis removes the noise problem and it is expected that this motive is not present in the SNC test. Nalatore et al. (2007) demonstrate that measurement noise can significantly impact Granger causality analysis.

Table 3. Symbolic Non Causality test applied to the 501 monthly returns for the 24 returns.

→	GSPC	NASDAQ	AA	AXP	BA	CAT	CVX	DD	DE	DIS	GE	GT	HPQ	IBM	JNJ	KO	MCD	MMM	MRK	PFE	PG	WFC	WMT	XOM
GSPC	6.06	**14.90***	2.85	5.74	0.98	1.92	1.56	**18.58****	1.27	5.63	5.38	7.62	3.90	0.58	9.45	11.73	4.80	3.39	1.59	10.18	9.09	3.90	2.42	1.99
NASDAQ	1.45	**17.35****	2.10	2.57	3.25	1.16	4.59	8.84	4.51	8.01	2.28	5.81	4.37	2.42	5.56	4.84	3.18	3.39	2.20	4.15	4.30	6.82	5.49	2.96
AA	4.66	4.87	12.77	7.62	3.25	6.32	1.66	**16.92****	6.93	4.30	1.88	5.52	9.67	2.60	11.40	5.27	3.97	1.34	11.04	10.14	7.29	6.42	4.30	3.14
AXP	6.53	**16.42***	6.42	4.08	3.21	2.64	5.52	7.90	6.75	8.37	5.67	1.48	6.53	5.31	3.14	0.87	12.20	2.93	1.99	2.06	5.92	11.37	6.82	9.09
BA	7.54	7.62	1.16	1.77	2.20	7.58	3.83	10.32	1.70	12.74	3.50	4.30	10.07	3.54	8.99	0.58	9.09	2.28	4.66	2.06	5.56	5.85	7.69	3.43
CAT	1.23	6.93	3.68	3.54	4.84	3.03	5.88	10.97	3.43	7.47	2.35	3.54	7.00	4.66	4.12	5.41	8.01	1.16	2.71	3.18	1.37	3.18	3.76	8.30
CVX	**18.36****	**13.82***	3.32	5.05	2.64	1.70	1.95	**13.39***	2.10	**17.93****	2.82	7.07	6.17	2.02	4.04	0.51	5.13	2.82	1.74	4.04	10.00	1.45	7.33	3.79
DD	6.39	8.55	1.99	1.27	2.49	3.68	2.42	8.77	4.33	8.81	1.77	3.07	12.70	5.27	7.33	2.71	3.76	11.62	6.57	7.04	3.29	6.39	2.49	5.99
DE	6.75	9.24	2.46	2.17	12.95	7.43	1.45	3.18	5.05	0.62	2.10	1.01	3.61	1.88	2.75	4.66	2.89	2.75	5.27	4.22	2.13	2.46	5.16	2.89
DIS	1.01	6.53	0.91	6.10	1.27	0.91	2.67	6.32	0.76	2.17	2.78	2.93	4.30	5.05	5.88	3.03	8.77	2.71	1.77	1.99	4.80	5.70	5.49	5.74
GE	1.09	4.69	1.30	3.11	4.01	0.80	4.04	5.67	1.12	9.20	6.82	7.90	2.28	2.02	8.59	3.07	0.11	1.70	0.33	4.22	2.06	0.62	4.51	6.10
GT	2.28	7.00	2.46	1.56	10.14	5.16	3.32	13.31	1.70	1.84	0.94	6.75	6.14	1.56	6.79	0.80	3.90	1.81	3.00	1.70	1.84	0.65	4.73	8.91
HPQ	4.51	**14.76***	1.37	5.20	4.87	0.80	4.48	10.93	2.24	1.88	3.43	6.68	1.70	4.73	7.00	3.76	2.17	2.60	1.63	7.11	12.99	1.45	2.85	4.80
IBM	2.49	6.93	3.07	2.78	2.42	1.05	3.00	4.37	5.20	8.26	1.27	2.28	3.36	3.39	8.59	4.77	2.46	3.86	3.72	5.09	6.61	1.59	5.99	3.32
JNJ	1.52	3.58	7.33	4.26	1.01	3.07	2.49	2.96	1.56	**14.40***	2.20	4.22	3.65	2.28	4.30	4.55	12.20	0.58	4.26	1.52	7.11	3.21	7.58	1.30
KO	4.37	6.53	4.19	0.65	3.32	5.23	5.02	6.32	1.88	**14.90***	8.81	5.96	2.67	4.69	3.21	1.88	3.50	3.79	4.12	4.19	4.59	1.92	6.03	1.16
MCD	4.62	9.96	2.96	2.24	6.28	6.39	4.77	9.82	3.90	1.74	9.38	7.65	8.84	1.88	1.37	2.06	7.65	1.09	3.79	3.32	3.97	2.82	6.28	2.38
MMM	1.16	8.84	3.29	1.99	**16.42****	2.10	1.63	3.43	1.63	9.53	3.03	5.13	3.65	1.45	3.29	4.30	2.13	6.24	6.32	1.09	3.65	2.60	5.20	10.50
MRK	7.29	2.78	8.77	2.64	4.91	7.15	1.63	6.57	0.76	4.77	4.12	1.41	0.91	3.18	3.21	2.64	8.12	4.37	12.48	4.51	**13.82***	1.23	0.22	1.27
PFE	0.91	11.87	11.33	2.78	2.60	3.97	2.28	5.88	3.65	3.03	4.44	0.83	2.42	2.49	2.06	3.94	0.47	6.79	2.93	2.71	4.15	4.62	1.74	2.64
PG	6.93	7.43	2.89	**14.51***	2.64	4.98	4.95	6.97	5.63	10.46	2.17	6.93	1.34	4.95	8.34	**14.47***	6.93	3.61	3.76	4.91	**14.61***	2.28	5.05	3.29
WFC	2.75	9.56	3.00	3.94	6.46	4.33	4.73	8.77	6.79	7.51	4.37	5.45	4.33	2.20	3.43	2.67	2.75	2.53	1.23	3.76	3.14	2.53	6.86	2.17
WMT	1.09	1.34	2.71	6.50	3.03	3.54	1.09	3.14	2.31	1.92	1.12	2.31	3.97	6.39	1.99	7.36	2.71	5.45	**16.78****	3.68	1.12	5.67	**21.32*****	5.52
XOM	10.83	6.53	2.28	0.98	3.03	2.60	3.36	7.15	7.40	8.01	5.81	4.69	3.94	6.57	2.57	1.23	2.49	1.34	3.58	2.42	2.46	4.30	1.19	2.82

Note: Based on the US stock returns. * Significance at the 0.10 level corresponding to the statistic value 13.36. ** Significance at the 0.05 level corresponding to the statistic value 15.51. *** Significance at the level 0.01, corresponding to the statistic 20.09.

Table 4.Granger Non Causality test applied to the 501 monthly returns for the 24 returns.

→	GSPC	NASDAQ	AA	AXP	BA	CAT	CVX	DD	DE	DIS	GE	GT	HPQ	IBM	JNJ	KO	MCD	MMM	MRK	PFE	PG	WFC	WMT	XOM
GSPC	0.335	0.206	0.124	0.083*	0.023**	0.649	0.231	0.662	0.463	0.003***	0.056**	0.435	0.026**	0.448	0.402	0.454	0.534	0.997	0.296	0.658	0.955	0.451	0.336	0.020**
NASDAQ	0.032**	0.895	0.127	0.008***	0.005***	0.924	0.175	0.296	0.198	0.000***	0.005***	0.722	0.002***	0.483	0.152	0.528	0.142	0.562	0.496	0.555	0.311	0.146	0.232	0.056*
AA	0.682	0.107	0.192	0.480	0.015**	0.603	0.241	0.681	0.411	0.353	0.155	0.386	0.230	0.558	0.099*	0.730	0.757	0.810	0.762	0.363	0.612	0.812	0.718	0.206
AXP	0.121	0.082**	0.035**	0.673	0.109	0.833	0.591	0.342	0.786	0.002***	0.049**	0.376	0.025**	0.951	0.532	0.047**	0.189	0.799	0.178	0.175	0.050**	0.014**	0.963	0.192
BA	0.418	0.422	0.817	0.945	0.157	0.420	0.142	0.567	0.867	0.650	0.889	0.027**	0.318	0.988	0.537	0.673	0.199	0.706	0.569	0.852	0.411	0.684	0.368	0.089*
CAT	0.976	0.591	0.638	0.468	0.027**	0.686	0.360	0.466	0.552	0.251	0.474	0.489	0.128	0.059	0.796	0.498	0.676	0.242	0.810	0.500	0.745	0.330	0.599	0.113
CVX	0.012**	0.012**	0.604	0.378	0.904	0.259	0.676	0.213	0.648	0.689	0.728	0.171	0.712	0.934	0.319	0.596	0.489	0.139	0.601	0.927	0.329	0.991	0.684	0.640
DD	0.337	0.448	0.103	0.019	0.047**	0.809	0.173	0.529	0.911	0.044**	0.395	0.281	0.015**	0.003*	0.808	0.034**	0.296	0.404	0.145	0.888	0.687	0.496	0.045**	0.034**
DE	0.723	0.531	0.132	0.506	0.199	0.063	0.417	0.524	0.105	0.199	0.422	0.338	0.167	0.237	0.275	0.300	0.646	0.768	0.596	0.072*	0.884	0.234	0.661	0.329
DIS	0.600	0.952	0.361	0.662	0.397	0.425	0.776	0.145	0.339	0.739	0.158	0.854	0.028**	0.565	0.816	0.129	0.475	0.721	0.272	0.901	0.688	0.707	0.016**	0.120
GE	0.754	0.576	0.060*	0.017**	0.115	0.551	0.667	0.777	0.430	0.021**	0.492	0.142	0.092*	0.793	0.989	0.652	0.435	0.503	0.401	0.999	0.794	0.051*	0.847	0.011**
GT	0.339	0.509	0.162	0.122	0.049**	0.693	0.109	0.204	0.885	0.348	0.081*	0.104	0.080*	0.141	0.941	0.548	0.141	0.711	0.701	0.436	0.343	0.043**	0.196	0.266
HPQ	0.337	0.025**	0.897	0.594	0.013**	0.425	0.476	0.606	0.899	0.127	0.198	0.092*	0.507	0.914	0.049**	0.730	0.646	0.817	0.729	0.384	0.033**	0.855	0.365	0.233
IBM	0.396	0.230	0.456	0.552	0.016**	0.274	0.509	0.738	0.522	0.483	0.923	0.744	0.885	0.565	0.503	0.700	0.096*	0.993	0.830	0.436	0.890	0.203	0.697	0.941
JNJ	0.999	0.468	0.092*	0.736	0.654	0.219	0.648	0.247	0.733	0.414	0.900	0.431	0.257	0.870	0.443	0.139	0.742	0.758	0.428	0.314	0.082*	0.996	0.032**	0.468
KO	0.532	0.186	0.020**	0.597	0.824	0.924	0.367	0.031**	0.874	0.327	0.419	0.785	0.044**	0.428	0.253	0.866	0.268	0.173	0.311	0.870	0.052*	0.986	0.138	0.417
MCD	0.624	0.958	0.294	0.669	0.632	0.767	0.395	0.250	0.610	0.866	0.940	0.944	0.319	0.253	0.159	0.033**	0.737	0.651	0.774	0.908	0.071*	0.910	0.227	0.846
MMM	0.909	0.973	0.424	0.508	0.034**	0.750	0.762	0.510	0.695	0.968	0.106	0.701	0.239	0.673	0.746	0.974	0.851	0.717	0.112	0.829	0.563	0.042**	0.281	0.233
MRK	0.334	0.505	0.102	0.663	0.069*	0.096*	0.111	0.316	0.392	0.745	0.758	0.689	0.376	0.662	0.881	0.370	0.127	0.362	0.967	0.181	0.316	0.342	0.880	0.787
PFE	0.700	0.641	0.318	0.247	0.987	0.462	0.144	0.853	0.832	0.244	0.133	0.955	0.544	0.745	0.254	0.586	0.937	0.820	0.154	0.386	0.110	0.426	0.401	0.653
PG	0.947	0.222	0.297	0.904	0.848	0.355	0.239	0.771	0.696	0.501	0.673	0.019**	0.411	0.377	0.504	0.441	0.493	0.144	0.103	0.686	0.593	0.930	0.268	0.754
WFC	0.512	0.221	0.697	0.267	0.433	0.769	0.626	0.940	0.133	0.250	0.234	0.953	0.009***	0.774	0.301	0.292	0.585	0.423	0.107	0.416	0.043**	0.870	0.449	0.025**
WMT	0.247	0.309	0.058*	0.287	0.018**	0.548	0.626	0.311	0.835	0.043**	0.032**	0.781	0.157	0.043**	0.169	0.229	0.004***	0.009***	0.024**	0.531	0.328	0.523	0.844	0.011**
XOM	0.040**	0.241	0.503	0.286	0.415	0.131	0.405	0.067*	0.092*	0.759	0.753	0.548	0.404	0.908	0.245	0.965	0.245	0.394	0.623	0.790	0.244	0.403	0.888	0.932

Note: Based on the US stock returns. The significance level of the test is presented in the matrix. * Significance at the 0.10 level. ** Significance at the 0.05. *** Significance at the level 0.

Table 5. Symbolic Non Causality test applied to the 2178 weekly returns for the 24 returns.

→	GSPC	NASDAQ	AA	AXP	BA	CAT	CVX	DD	DE	DIS	GE	GT	HPQ	IBM	JNJ	KO	MCD	MMM	MRK	PFE	PG	WFC	WMT	XOM
GSPC	16.03**	12.36	11.81	16.96**	16.87**	8.39	11.27	2.36	1.11	21.99***	16.16**	9.92	4.04	8.47	11.63	16.16**	15.35*	8.24	3.63	22.89***	7.76	14.95*	5.48	5.09
NASDAQ	13.98*	9.19	4.23	6.78	3.84	9.42	7.96	4.02	0.39	11.05	7.47	5.48	9.90	9.58	8.03	8.49	6.96	9.73	7.51	11.89	6.64	2.05	5.17	1.32
AA	9.31	7.67	0.50	7.82	2.53	5.98	0.39	6.79	2.69	10.73	7.56	14.04*	5.97	3.82	5.69	7.39	6.02	19.76**	8.31	7.99	8.34	3.89	9.61	2.15
AXP	9.01	9.40	8.46	12.16	4.69	6.72	6.25	10.09	5.23	12.65	10.37	3.91	3.64	4.38	1.61	9.71	3.20	11.07	2.51	13.62*	7.22	19.35**	1.86	2.38
BA	3.56	3.56	4.33	9.73	10.40	9.88	1.44	3.20	5.23	9.28	1.65	5.28	5.46	1.70	1.13	11.23	6.25	3.62	2.57	9.01	6.30	5.45	3.52	3.76
CAT	9.21	3.65	2.92	5.62	6.74	10.64	5.17	8.09	2.17	5.77	2.28	3.98	2.15	2.35	9.17	7.79	2.69	5.26	3.01	6.73	12.22	19.55**	8.56	2.87
CVX	14.14*	7.56	2.74	4.58	6.06	2.78	8.73	5.92	5.03	15.94**	4.78	5.04	2.04	4.63	9.66	15.47*	2.15	3.50	11.22	14.65*	14.16*	6.21	4.71	1.78
DD	10.72	11.96	6.48	8.40	2.29	1.94	0.64	5.57	6.39	11.30	0.64	2.90	5.99	11.40	10.94	6.35	12.58	12.05	5.78	9.58	10.71	4.48	9.80	4.63
DE	6.84	5.31	2.93	6.33	0.88	0.21	8.53	0.72	0.26	1.49	5.57	4.96	3.60	7.36	13.17	13.92*	0.87	8.10	3.94	4.88	7.75	4.74	8.20	1.71
DIS	7.74	8.45	4.36	4.65	4.96	4.60	2.89	3.74	5.12	15.55**	3.38	1.45	0.63	0.66	9.92	3.28	9.03	11.48	0.09	2.08	3.99	6.67	5.55	6.86
GE	17.20**	15.03*	0.73	5.55	0.56	6.14	6.18	0.64	1.32	17.01**	9.08	4.78	7.80	3.27	8.92	4.85	5.42	9.68	4.84	4.45	4.45	14.11*	0.88	2.55
GT	1.50	3.14	4.40	0.53	9.95	3.12	5.16	15.04*	1.55	7.23	0.53	7.05	3.08	3.65	8.04	7.87	9.99	6.95	3.07	10.42	12.67	13.88*	4.05	1.21
HPQ	10.49	6.27	1.26	7.19	1.75	5.27	6.05	3.67	3.52	5.64	10.56	1.54	4.13	11.24	11.16	3.10	6.13	5.18	3.29	12.68	4.47	15.82**	5.51	2.15
IBM	7.22	5.38	4.22	1.24	2.50	0.76	7.16	4.94	4.23	5.91	5.35	9.90	9.65	2.68	1.50	5.73	1.94	7.61	4.26	10.11	4.34	10.04	2.25	2.19
JNJ	4.72	5.80	2.60	4.62	6.49	8.60	2.98	4.57	2.29	11.52	2.55	1.91	2.02	6.48	9.40	2.12	7.36	6.31	10.50	6.81	3.95	3.86	9.71	3.33
KO	13.62*	19.08**	7.34	5.14	7.75	9.92	4.18	2.94	1.97	14.95*	6.61	2.55	8.57	10.88	0.54	5.95	4.77	17.32**	7.72	11.09	2.43	13.81*	9.67	5.04
MCD	2.20	10.00	7.11	3.43	3.56	3.24	5.13	4.88	4.20	5.28	2.27	1.60	2.98	2.45	1.53	3.51	0.32	6.53	2.20	2.25	1.86	5.12	7.59	1.75
MMM	6.14	2.74	4.66	2.81	7.16	3.34	3.13	3.90	5.23	6.95	0.64	5.82	4.04	6.99	6.28	13.79*	8.54	14.71*	8.59	10.53	9.20	1.29	2.17	3.46
MRK	4.26	9.37	6.09	2.78	5.35	6.84	5.50	10.64	5.35	5.40	0.35	0.38	1.88	1.98	3.00	2.62	3.80	6.59	3.65	2.21	5.95	3.40	5.68	3.03
PFE	9.44	8.73	6.88	17.87**	3.27	7.81	5.08	5.76	2.96	0.50	6.81	2.50	3.10	5.45	3.51	6.19	2.44	6.90	1.07	9.46	2.61	18.47**	20.25***	0.32
PG	9.19	0.97	16.94**	3.34	3.14	5.20	4.99	4.66	0.46	13.66*	4.03	2.26	8.18	6.40	5.88	3.76	3.12	10.57	3.38	18.57**	8.73	2.97	6.75	3.35
WFC	7.82	11.69	8.90	4.67	3.63	4.86	3.56	4.07	5.42	2.86	6.84	7.42	5.90	5.97	3.34	8.14	3.77	4.99	3.17	1.56	6.66	25.71***	10.55	3.52
WMT	8.30	4.30	3.96	5.40	3.96	4.76	1.45	6.86	1.04	3.68	21.01***	2.92	7.74	11.36	1.58	12.20	7.48	4.80	2.48	18.04**	1.96	41.82***	218.3***	3.00
XOM	14.15*	10.08	14.61*	4.38	5.23	10.01	6.32	4.38	5.62	18.74**	0.73	11.02	2.36	7.77	11.03	10.46	4.02	2.30	4.99	13.83*	6.71	7.81	10.40	3.65

Note: Based on the US stock returns. * Significance at the 0.10 level corresponding to the statistic value 13.36. ** Significance at the 0.05 level corresponding to the statistic value 15.51. *** Significance at the level 0.01, corresponding to the statistic 20.09.

Table 6.Granger Non Causality test applied to the 2178weekly returns for the 24 returns.

→	GSPC	NASDAQ	AA	AXP	BA	CAT	CVX	DD	DE	DIS	GE	GT	HPQ	IBM	JNJ	KO	MCD	MMM	MRK	PFE	PG	WFC	WMT	XOM
GSPC	0.543	0.409	0.295	0.716	0.691	0.003***	0.354	0.674	0.415	0.248	0.162	0.827	0.254	0.883	0.001***	0.233	0.623	0.223	0.013**	0.386	0.138	0.365	0.808	0.552
NASDAQ	0.326	0.845	0.101	0.089*	0.838	0.036**	0.199	0.887	0.237	0.115	0.098*	0.973	0.068*	0.766	0.002***	0.242	0.827	0.674	0.043**	0.454	0.044**	0.733	0.936	0.329
AA	0.518	0.380	0.737	0.544	0.725	0.020**	0.609	0.126	0.453	0.827	0.699	0.906	0.242	0.832	0.000***	0.438	0.761	0.031**	0.000***	0.026**	0.132	0.910	0.971	0.459
AXP	0.001***	0.010***	0.005***	0.678	0.497	0.000***	0.004***	0.000***	0.019**	0.000***	0.002***	0.315	0.619	0.252	0.945	0.047**	0.060*	0.002***	0.884	0.310	0.571	0.001*	0.858	0.060*
BA	0.292	0.099*	0.978	0.047**	0.354	0.147	0.689	0.629	0.189	0.026**	0.109	0.358	0.529	0.969	0.819	0.842	0.153	0.016**	0.904	0.099*	0.841	0.237	0.395	0.981
CAT	0.810	0.823	0.207	0.273	0.636	0.659	0.506	0.219	0.530	0.456	0.150	0.308	0.764	0.758	0.159	0.255	0.983	0.201	0.021**	0.031**	0.089*	0.045*	0.177	0.090*
CVX	0.468	0.765	0.141	0.209	0.674	0.191	0.854	0.369	0.371	0.059*	0.753	0.523	0.929	0.874	0.131	0.440	0.875	0.694	0.012**	0.027**	0.230	0.638	0.673	0.004*
DD	0.575	0.490	0.609	0.410	0.854	0.031**	0.088*	0.367	0.389	0.016**	0.297	0.134	0.812	0.877	0.011**	0.141	0.640	0.171	0.011**	0.169	0.233	0.022*	0.603	0.295
DE	0.933	0.237	0.111	0.968	0.602	0.012**	0.224	0.077*	0.314	0.720	0.588	0.820	0.464	0.526	0.199	0.670	0.365	0.028**	0.386	0.660	0.998	0.389	0.985	0.038**
DIS	0.176	0.066*	0.044**	0.278	0.712	0.090*	0.113	0.781	0.882	0.673	0.063*	0.309	0.168	0.977	0.456	0.100	0.047**	0.421	0.528	0.737	0.753	0.064*	0.132	0.996
GE	0.065*	0.030**	0.070*	0.149	0.894	0.263	0.227	0.402	0.604	0.027**	0.343	0.521	0.350	0.693	0.232	0.281	0.106	0.779	0.217	0.638	0.768	0.113	0.865	0.453
GT	0.065*	0.392	0.002***	0.128	0.249	0.011**	0.281	0.003***	0.514	0.194	0.062	0.305	0.235	0.339	0.138	0.583	0.183	0.041**	0.555	0.935	0.965	0.024**	0.323	0.102
HPQ	0.168	0.390	0.009***	0.212	0.533	0.004***	0.001***	0.164	0.071	0.422	0.136	0.192	0.983	0.711	0.006***	0.718	0.870	0.055*	0.147	0.884	0.194	0.577	0.811	0.011**
IBM	0.744	0.531	0.232	0.683	0.849	0.071*	0.352	0.504	0.600	0.837	0.390	0.789	0.684	0.851	0.306	0.108	0.310	0.002***	0.990	0.395	0.845	0.773	0.623	0.133
JNJ	0.021**	0.024**	0.624	0.015**	0.961	0.044**	0.251	0.034**	0.893	0.283	0.129	0.495	0.229	0.301	0.471	0.013**	0.108	0.000***	0.085*	0.007***	0.097*	0.028**	0.089*	0.617
KO	0.958	0.168	0.924	0.984	0.649	0.419	0.571	0.889	0.294	0.017**	0.308	0.541	0.212	0.992	0.653	0.139	0.059*	0.152	0.920	0.473	0.228	0.036**	0.299	0.201
MCD	0.006***	0.034**	0.312	0.008***	0.246	0.001***	0.006***	0.000***	0.230	0.000***	0.020	0.016**	0.025**	0.239	0.089*	0.001***	0.335	0.100*	0.133	0.047**	0.011**	0.015**	0.167	0.475
MMM	0.396	0.909	0.819	0.269	0.200	0.020**	0.678	0.501	0.169	0.107	0.974	0.536	0.750	0.590	0.060*	0.265	0.361	0.889	0.813	0.848	0.917	0.104	0.883	0.604
MRK	0.450	0.611	0.325	0.791	0.294	0.541	0.532	0.435	0.250	0.560	0.134	0.803	0.863	0.483	0.839	0.259	0.649	0.447	0.215	0.245	0.026**	0.023**	0.392	0.874
PFE	0.296	0.626	0.844	0.350	0.852	0.368	0.226	0.164	0.769	0.230	0.191	0.672	0.662	0.256	0.249	0.082*	0.235	0.006***	0.379	0.946	0.109	0.006***	0.527	0.242
PG	0.052*	0.633	0.592	0.221	0.317	0.320	0.676	0.090*	0.790	0.838	0.693	0.433	0.819	0.337	0.006***	0.847	0.973	0.769	0.480	0.570	0.903	0.643	0.478	0.251
WFC	0.636	0.514	0.748	0.875	0.961	0.102	0.580	0.103	0.756	0.095*	0.313	0.813	0.294	0.879	0.493	0.840	0.326	0.567	0.041**	0.897	0.916	0.856	0.254	0.350
WMT	0.180	0.037**	0.087	0.584	0.104	0.012**	0.891	0.499	0.293	0.001***	0.029**	0.963	0.600	0.850	0.686	0.255	0.000***	0.017**	0.295	0.955	0.288	0.015**	0.940	0.972
XOM	0.165	0.247	0.117	0.476	0.831	0.943	0.777	0.504	0.239	0.531	0.633	0.366	0.918	0.865	0.069*	0.967	0.775	0.948	0.092*	0.017**	0.319	0.627	0.128	0.890

Note: Based on the US stock returns. The significance level of the test is presented in the matrix. * Significance at the 0.10 level. ** Significance at the 0.05. *** Significance at the level 0.01.

6. Conclusion

Causality is still a discussed topic at a philosophical level, however its practical application in different field of the sciences it is alive. In particular, according to Granger there is a characteristic of causality having the major practical application, time. In fact, he said that thinking of causality makes sense when temporal predetermination is considered. In this sense, the cause chronologically precedes the effect.

The Granger non-causality test is among the most applied tool testing causality and it is widely applied in empirical investigation and is found in most of the econometric packages. However, the test may present a series of problems. At first, the test is designed under the VAR model, showing a good performance when detecting this particular kind of causality but showing difficult detecting nonlinear causality. Secondly, in spite of some nonlinear generalization of Granger test, these tests are still based on determined nonlinear models and detection of causal nonlinearity is linked to the underlying specification model form. Finally, as mentioned before some authors remark that when working with empirical time series, they are contaminated by noise sometime generating spurious causality or not allowing detecting the true causality.

In the present work, it was present a first approach to a nonparametric non-causality test based on the symbolic time series analysis. The idea is to develop a complementary test to the Granger non-causality, showing strengths in the points where the Granger test is weak. In this sense it was shown that the proposed SNC test present a good performance detecting nonlinear processes, in particular the chaotic processes and it is expected that since it applied the symbolic time series analysis the problem related with the spurious causality due to noise should be mitigated.

The conducted experiments show that the novel test presents a good performance detecting nonlinear and chaotic processes such as models with exponential components, the NLAR model and the chaotic Lorenz map. These models are not detected by the Granger test in the experiments.

The application of both tests to the US financial data seems to suggest that both tests may detect different kind of causality. Even though there is no the objective of the present paper to go deeper on the very cause of these differences, it can be mentioned some reasons. For instance, a same noise can be affecting a number of financial time series provoking spurious causalities or could be nonlinear causalities which are not detected by Granger linear test. For this reason, it is suggested to apply both tests as complementary.

As mentioned in Risso (2014), the symbolic time series analysis seems to have potential and it should be developed generating practical tools for the econometric analysis. Further research should be focus on to go deeper in this test trying to improve its potential. On the other hand, a second line of research should be to find new practical application of the symbolic analysis.

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